ON THE REQUESTS OF DESCENDANTS AT YULETIDE

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Abstract. We define the notion of a *quasi-Christmas list*, which is a generalisation of the well-known Christmas list. An important property of this structure is given, with reference to a particularly relevant worked example.

Since the late 1990's, the author has developed a number of novel ways for the transmission of Christmas lists. The inspiration for this article came when the author was requested to repeat this process this year. For the reader's benefit, we recall the definition of a Christmas list:

Definition 1. A *Christmas list* L is a finite set of pairs $(\text{gift}, i) \in \mathbb{G} \times I$, where $\mathbb{G}$ is the set of all possible gifts and $I = \{1, \ldots, n\}$ where $n \in 5\mathbb{N}$. We call n the *length* of a Christmas list. Moreover, such an L comes equipped with a partially defined function $\theta : L \dashrightarrow \mathbb{P}$ defined by the recipient of the list, where $\mathbb{P}$ is the set of all potential possessions of the Christmas list's author.

Informally, a Christmas list is an ordered collection of elements which are proposed new entries into the set of author's possessions.

While the above is a well-accepted definition which has motivated the creation of a number of interesting examples, we feel that the need for the length of a Christmas list to be divisible by five is a somewhat arbitrary restriction; we assume that this is simply a byproduct of the base 10 number system employed by the general population. With this in mind, we make the following definition:

Definition 2. A *quasi-Christmas list* L is defined in the same way as a Christmas list (Definition 1), but where the length n is now allowed to take values in $\mathbb{N}$.

While this is only a minor generalisation of the usual definition, we are now able to apply a wide variety of number-theoretic techniques when studying these objects. For example, we have the following remarkable result:

Theorem 3. *The length of a quasi-Christmas list can be four.*

The proof of this theorem is, somewhat surprisingly, reasonably clear. The diligent reader is encouraged to extract it from the following example:

Example 4. The author's quasi-Christmas list L for this year is

$$L = \left\{ \begin{array}{c} (\text{First gift}, 1) \\ (\text{Second gift}, 2) \\ (\text{Third gift}, 3) \\ (\text{Fourth gift}, 4) \end{array} \right\}.$$

The reader is left to define $\theta : L \dashrightarrow \mathbb{P}$ as they deem appropriate.

In the future, we hope to generalise the definition of a Christmas list to allow $n \in \mathbb{C}$, but this does not seem possible using our current techniques.

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